

Example: 3x2 MIMO system

$\downarrow$ $\downarrow$
 $r=3$ $t=2$
Rx ant Tx ant

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \underbrace{\begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}}_{\substack{H \\ 3 \times 2 \\ r > t}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix}$$

$$H = \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}$$

Zero Forcing Receiver,

$$= \underbrace{(H^H H)^{-1}}_{\substack{\nearrow \\ \text{Let us evaluate} \\ \text{This matrix}}} H^H \bar{y}$$

$$H^H H = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ 1 & 3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 17 \\ 17 & 22 \end{bmatrix}$$

$$(H^H H)^{-1} = \frac{1}{21 \times 22 - 17^2} \begin{bmatrix} 22 & -17 \\ -17 & 21 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1272 & -0.0983 \\ -0.0983 & 0.1214 \end{bmatrix}$$

$$(H^H H)^{-1} H^H$$

$$= \begin{bmatrix} 0.1272 & -0.0983 \\ -0.0983 & 0.1214 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 3 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -0.0405 & -0.1676 & 0.3121 \\ 0.1676 & 0.2659 & -0.1503 \end{bmatrix}$$

$$(H^H H)^{-1} H^H$$

Pseudo inverse
of H .

$$\hat{x} = (H^H H)^{-1} H^H \bar{y}$$

$$\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} -0.0405 & -0.1676 & 0.3121 \\ 0.1676 & 0.2659 & -0.1503 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{cases} \hat{x}_1 = -0.0405 y_1 - 0.1676 y_2 + 0.3121 y_3 \\ \hat{x}_2 = 0.1676 y_1 + 0.2659 y_2 - 0.1503 y_3 \end{cases}$$

→ Zero Forcing Receiver

BER of MIMO system

with ZF receiver.

consider matrix H $r \times t$

Transmit
symbol power = P
 $E\{|x_i|^2\} = P.$

With all elements IID
Complex Gaussian
average power = 1

→ IID Rayleigh
average power = 1

Bit error rate, for BPSK

With Zero Forcing (ZF) Receiver

= Receive antenna MRC system
with $L = r - t + 1$ antennas.

$$\text{BER} = 2L^{-1} \Gamma_L \cdot \left(\frac{1}{2 \text{SNR}} \right)^L$$

at high SNR ✓

with $L = r - t + 1$.

Example: Consider 3×2 MIMO system.

$$\text{SNR} = 25 \text{ dB}$$

ZF Receiver.

What is BER?

$$\text{Average BER} = 2^{L-1} C_L \cdot \left(\frac{1}{2 \text{SNR}} \right)^L$$

$$L = r - t + 1$$

$$= 3 - 2 + 1 = 2$$

$$\text{Avg BER} = 3 C_2 \cdot \left(\frac{1}{2 \text{SNR}} \right)^2 = \frac{3}{4} \cdot \frac{1}{\text{SNR}^2}$$

$$\text{SNR (dB)} = 25 \text{ dB}$$

$$10 \log_{10} \text{SNR} = 25$$

$$\text{SNR} = 10^{2.5} = 316.2278$$

$$\text{BER} = \frac{3}{4} \cdot \left(\frac{1}{10^{2.5}} \right)^2$$

$$= 0.75 \times 10^{-5}$$

$$\text{BER} = 7.5 \times 10^{-6}$$